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REVIEWS

Partial Differential Equations. By I. G. PETROVSKII. Iliffe Books Ltd, 1967. 410 pp. 50*s*.

Equations of the Mixed Type. By A. V. BITSADZE. Pergamon Press, 1964. 160 pp. 60*s*.

Russian mathematicians have made many important contributions to the theory of partial differential equations, and translations of Russian books on this subject are therefore always of interest. The two books under review are, respectively, a standard text at third-year undergraduate level, and a research monograph, both written by pure mathematicians.

Professor Petrovskii's own contribution to partial differential equation theory are outstanding, and well known. His book, which is based on a course of lectures, does not deal with these, but is an exposition of the classical theory, mainly concerned with linear second-order equations. It has run through several editions; a translation of the first edition was published by Interscience in 1954, and the present translation appears to have been made from the third (1961) edition. There is an introductory chapter, which includes the Cauchy–Kowalewski theorem, Homgren's uniqueness theorem, and the reduction of second-order equations with two independent variables to canonical form. Chapter 2, on hyperbolic equations, is divided into two parts; the first deals, in effect, with transient problems, while the second part gives a thorough account of the application of eigenfunction theory to oscillation problems. Chapter 3, on elliptic equations, is primarily devoted to Laplace's equation, and gives the theory of harmonic functions, and the solution of boundary value problems by the integral equation method. The book concludes with a short chapter on the diffusion equation, and a supplement (added in the third Russian edition) on finite difference methods. The mathematical approach is rigorous, but the motivation of the formal proofs is often explained heuristically, and the genesis of mathematical problems from physical questions is emphasized. Numerous references to recent work on partial differential equations are given; some of these are at a level considerably above that of the text.

The translation is adequate. It is to be regretted that, in references to Russian books (in particular, to two frequently quoted books by Sobolev), no attempt has been made to draw attention to translations of such works into English. On the other hand, the well-known paper by Courant, Friedrichs & Lewy, published in 1928 in Volume 100 of *Mathematische Annalen*, is referred to, on p. 136, in Russian translation.)

Like most texts on the subject, Petrovskii's book does not deal with equations of mixed type, which are elliptic in one region, and hyperbolic in an adjacent region. The theory of such equations, which are of course of particular interest in gas dynamics, is difficult, and still being developed, although the literature is now quite extensive. The main effort has so far been directed to the case of two

independent variables, the study of which was initiated by Tricomi's celebrated 1923 paper. Very few connected accounts have appeared so far, and Professor Bitsadze's monograph is therefore a welcome addition to the literature. It was published, in Russian, in 1959. There is an introductory chapter on the transformation of certain classes of equation of mixed type to normal form. This is followed by chapters on hyperbolic equations with a parabolic line, and on elliptic equations with a parabolic line. The last two chapters, which constitute a little over half of the book, deal, respectively, with Tricomi's problem, and with generalizations of this problem, such as Frankl's problem. The theory is developed mainly for the Tricomi equation

$$yu_{xx}+u_{yy} = 0$$

and for the Lavrent' ev–Bitsadze equation

$$u_{xx}\text{sign } y+u_{yy} = 0.$$

The treatment concentrates on the now classical method of reduction to singular integral equations, and on the author's own work, which is more function-theoretical. The Hilbert Space approach due to Friedrichs and Morawetz is touched on briefly; methods based on integral transforms are not included. There is an extensive bibliography. An interesting feature is the listing of unsolved problems at the end of each chapter. The reader should be warned that there are numerous misprints.

The approach to the subject is purely mathematical, with virtually no reference to physical problems, but this is a monograph that should prove of considerable interest to applied mathematicians, particularly to anyone engaged in theoretical work on transonic flow.

F. G. FRIEDLANDER

Theoretische Strömungslehre. By K. WIEGHARDT. Teubner, 1965. 226 pp. DM. 35.80.

Gasdynamik. By E. BECKER. Teubner, 1965. 248 pp. DM. 39.60.

These two volumes in the series 'Leitfäden der angewandten Mathematik und Mechanik' may be considered to be complementary to one another. Professor Wieghardt begins with the basic physical properties and mathematical equations of fluid flow. He then gives an excellent account of the theory of inviscid fluids, the development following normal lines, illustrating both the power and the limitations of the mathematical methods. There are sections dealing with potential flows past aerofoils and discontinuous flows, and on the potential flow of an ideal gas. The deficiencies of the results obtained by the classical methods are pointed out, comparisons with experimental work being given in particular instances, and the physical reasons for the differences are briefly described. At the end of the sections the author often indicates in a few sentences some of the extensions and refinements that have been attempted or need to be found to deal with various practical problems, and he likes to give examples of phenomena from and contrasts between different aspects of fluid flow, e.g. meteorology, water waves, flow of discrete particles. The final part of the text, concerned with viscous flows, occupies one-third of the book. Beginning with

exact solutions of the Navier–Stokes equations, slow flows and the Oseen approximation, the author goes on to develop boundary-layer theory, giving the standard treatment and results, describing in physical terms the separation phenomenon and the effects of separation and of turbulence on the resistance to motion, and making a few remarks on the numerical solution of the equations of laminar motion. There is a short account of the theory of hydrodynamical stability, including the problems of Poiseuille and Couette flow and Taylor flow between rotating cylinders. The last section deals with turbulent flows. The physical rôle of small and large eddies is briefly described, and then the theoretical treatment begins. The semi-empirical theories, which were developed for the flows of greatest practical importance are introduced; flows in tubes and along a plate are discussed, and then the classical examples of free turbulence, plane and round jets, are examined. Finally the basic ideas of the statistical theory of isotropic turbulence are given.

Professor Becker has deliberately omitted from his book the more usual aspects of subsonic and transonic flow, and concerns himself mostly with supersonic flow. In this way, within the space of 248 pages, he has succeeded in including an account of the effects of departures from ideal gas behaviour and of relaxation from thermodynamic equilibrium, so that the attention of students is drawn to the modifications of classical theory that are necessary before a study can be made of some of the most important modern applications. The sections on relaxation effects are indicated throughout by an asterisk, and these sections, together with others in small print, which provide very useful guides to extensions in the research literature of the previous introductory work, may be omitted on a first reading without spoiling the text. Professor Becker begins with a substantial discussion of thermodynamical principles, extended beyond what is normally expected of an introductory text-book in order to allow the newer topics to be discussed. He then goes on to derive the basic equations. About one-half of the book deals with inviscid flows, giving the basic material on steady and unsteady flows, expansion and shock waves and the method of characteristics, together with the modern extensions already mentioned and a fair number of references to original papers. The last chapter, of 50 pages, deals with viscous flows, with particular reference to the processes inside shock waves and to boundary-layer theory. There is an abundance of pen-and-ink figures to illustrate the material, and these are certainly helpful to the reader.

Both books are excellent texts for advanced students and have been very well prepared and printed.

D. C. Pack